***CHAPTER 1***

**INTRODUCTION TO CORPORATE FINANCE**

# Answers to Concept Questions

**1.** In the corporate form of ownership, the shareholders are the owners of the firm. The shareholders elect the directors of the corporation, who in turn appoint the firm’s management. This separation of ownership from control in the corporate form of organization is what causes agency problems to exist. Management may act in its own or someone else’s best interests, rather than those of the shareholders. If such events occur, they may contradict the goal of maximizing the share price of the equity of the firm.

**2.** Such organizations frequently pursue social or political missions, so many different goals are conceivable. One goal that is often cited is revenue minimization; i.e., provide whatever goods and services are offered at the lowest possible cost to society. A better approach might be to observe that even a not-for-profit business has equity. Thus, one answer is that the appropriate goal is to maximize the value of the equity.

**3.** Presumably, the current stock value reflects the risk, timing, and magnitude of all future cash flows, both short-term *and* long-term. If this is correct, then the statement is false.

**4.** An argument can be made either way. At the one extreme, we could argue that in a market economy, all of these things are priced. There is thus an optimal level of, for example, ethical and/or illegal behavior, and the framework of stock valuation explicitly includes these. At the other extreme, we could argue that these are non-economic phenomena and are best handled through the political process. A classic (and highly relevant) thought question that illustrates this debate goes something like this: “A firm has estimated that the cost of improving the safety of one of its products is $30 million. However, the firm believes that improving the safety of the product will only save $20 million in product liability claims. What should the firm do?”

**5.** The goal will be the same, but the best course of action toward that goal may be different because of differing social, political, and economic institutions.

**6.** The goal of management should be to maximize the share price for the current shareholders. If management believes that it can improve the profitability of the firm so that the share price will exceed $35, then they should fight the offer from the outside company. If management believes that this bidder, or other unidentified bidders, will actually pay more than $35 per share to acquire the company, then they should still fight the offer. However, if the current management cannot increase the value of the firm beyond the bid price, and no other higher bids come in, then management is not acting in the interests of the shareholders by fighting the offer. Since current managers often lose their jobs when the corporation is acquired, poorly monitored managers have an incentive to fight corporate takeovers in situations such as this.

**7.** We would expect agency problems to be less severe in other countries, primarily due to the relatively small percentage of individual ownership. Fewer individual owners should reduce the number of diverse opinions concerning corporate goals. The high percentage of institutional ownership might lead to a higher degree of agreement between owners and managers on decisions concerning risky projects. In addition, institutions may be better able to implement effective monitoring mechanisms on managers than can individual owners, based on the institutions’ deeper resources and experiences with their own management.

**8.** The increase in institutional ownership of stock in the United States and the growing activism of these large shareholder groups may lead to a reduction in agency problems for U.S. corporations and a more efficient market for corporate control. However, this may not always be the case. If the managers of the mutual fund or pension plan are not concerned with the interests of the investors, the agency problem could potentially remain the same, or even increase, since there is the possibility of agency problems between the fund and its investors.

**9.** How much is too much? Who is worth more, Larry Ellison or Tiger Woods? The simplest answer is that there is a market for executives just as there is for all types of labor. Executive compensation is the price that clears the market. The same is true for athletes and performers. Having said that, one aspect of executive compensation deserves comment. A primary reason executive compensation has grown so dramatically is that companies have increasingly moved to stock-based compensation. Such movement is obviously consistent with the attempt to better align stockholder and management interests. In recent years, stock prices have soared, so management has cleaned up. It is sometimes argued that much of this reward is due to rising stock prices in general, not managerial performance. Perhaps in the future, executive compensation will be designed to reward only differential performance, i.e., stock price increases in excess of general market increases.

**10.** Maximizing the current share price is the same as maximizing the future share price at any future period. The value of a share of stock depends on all of the future cash flows of company. Another way to look at this is that, barring large cash payments to shareholders, the expected price of the stock must be higher in the future than it is today. Who would buy a stock for $100 today when the share price in one year is expected to be $80?

***CHAPTER 4***

**DISCOUNTED CASH FLOW VALUATION**

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Assuming positive cash flows and interest rates, the future value increases and the present value decreases.

**2.** Assuming positive cash flows and interest rates, the present value will fall and the future value will rise.

**3.** The better deal is the one with equal installments.

**4.** Yes, they should. APRs generally don’t provide the relevant rate. The only advantage is that they are easier to compute, but, with modern computing equipment, that advantage is not very important.

**5.** A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue.

**6.** It’s a reflection of the time value of money. TMCC gets to use the $24,099 immediately. If TMCC uses it wisely, it will be worth more than $100,000 in thirty years.

**7.** This will probably make the security less desirable. TMCC will only repurchase the security prior to maturity if it is to its advantage, i.e. interest rates decline. Given the drop in interest rates needed to make this viable for TMCC, it is unlikely the company will repurchase the security. This is an example of a “call” feature. Such features are discussed at length in a later chapter.

**8.** The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the $100,000? Thus, our answer does depend on who is making the promise to repay.

**9.** The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.

**10.** The price would be higher because, as time passes, the price of the security will tend to rise toward $100,000. This rise is a reflection of the time value of money. As time passes, the time until receipt of the $100,000 grows shorter, and the present value rises. In 2019, the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or TMCC’s financial position could deteriorate. Either event would tend to depress the security’s price.

**Solutions to Questions and Problems**

*NOTE: All-end-of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

 *Basic*

**1.** The time line for the cash flows is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $7,800 |  |  |  |  |  |  |  |  |  | FV |

 The simple interest per year is:

 $7,800 × .077 = $600.60

 So, after 10 years, you will have:

 $600.60 × 10 = $6,006 in interest.

 The total balance will be $7,800 + 6,006 = $13,806

 With compound interest, we use the future value formula:

 FV = PV(1 + *r*)*t*

 FV = $7,800(1.077)10 = $16,377.65

 The difference is:

 $16,377.65 – 13,806 = $2,571.65

**2.** To find the FV of a lump sum, we use:

 FV = PV(1 + *r*)*t*

 *a.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,250 |  |  |  |  |  |  |  |  |  | FV |

 FV = $1,250(1.05)10 = $2,036.12

 *b.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,250 |  |  |  |  |  |  |  |  |  | FV |

 FV = $1,250(1.10)10 = $3,242.18

 *c.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 20 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,250 |  |  |  |  |  |  |  |  |  | FV |

 FV = $1,250(1.05)20 = $3,316.62

 *d.* Because interest compounds on the interest already earned, the interest earned in part *c* is more than twice the interest earned in part *a*. With compound interest, future values grow exponentially.

**3.** To find the PV of a lump sum, we use:

 PV = FV/(1 + *r)t*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 6 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV |  |  |  |  |  |  |  |  |  | $13,827 |

PV = $13,827/(1.07)6 = $9,213.51

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 11 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV |  |  |  |  |  |  |  |  |  | $43,852 |

 PV = $43,852/(1.15)11 = $9,425.69

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 19 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV |  |  |  |  |  |  |  |  |  | $725,380 |

 PV = $725,380/(1.11)19 = $99,868.60

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 29 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV |  |  |  |  |  |  |  |  |  | $590,710 |

 PV = $590,710/(1.18)29 = $4,861.79

**4.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

 FV = PV(1 + *r*)*t*

 Solving for *r*, we get:

 *r* = (FV/PV)1/*t* – 1

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 4 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$189 |  |  |  |  |  |  |  |  |  | $287 |

 FV = $287 = $189(1 + *r*)4; *r* = ($287/$189)1/4 – 1 = .1101, or 11.01%

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 8 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$410 |  |  |  |  |  |  |  |  |  | $887 |

 FV = $887 = $410(1 + *r*)8; *r* = ($887/$410)1/8 – 1 = .1013, or 10.13%

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 14 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$51,700 |  |  |  |  |  |  |  |  |  | $152,184 |

 FV = $152,184 = $51,700(1 + *r*)14; *r* = ($152,184/$51,700)1/14 – 1 = .0802, or 8.02%

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 27 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$21,400 |  |  |  |  |  |  |  |  |  | $538,600 |

 FV = $538,600 = $21,400(1 + *r*)27; *r* = ($538,600/$21,400)1/27 – 1 = .1269, or 12.69%

**5.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

 FV = PV(1 + *r*)*t*

 Solving for *t*, we get:

 *t* = ln(FV/PV)/ln(1 + *r*)

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | ? |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$625 |  |  |  |  |  |  |  |  |  | $1,104 |

 FV = $1,104 = $625(1.07)*t*; *t* = ln($1,104/$625)/ln 1.07 = 8.41 years

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | ? |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$810 |  |  |  |  |  |  |  |  |  | $5,275 |

 FV = $5,275 = $810(1.112)*t*; *t* = ln($5,275/$810)/ln 1.12 = 16.53 years

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | ? |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$16,500 |  |  |  |  |  |  |  |  |  | $245,830 |

 FV = $245,830 = $16,500(1.17)*t*; *t* = ln($245,830/$16,500)/ln 1.17 = 17.21 years

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | ? |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$21,500 |  |  |  |  |  |  |  |  |  | $215,000 |

 FV = $215,000 = $21,500(1.08)*t*; *t* = ln($215,000/$21,500)/ln 1.08 = 29.92 years

**6.** To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

 FV = PV(1 + *r*)*t*

 Solving for *t*, we get:

 *t* = ln(FV/PV)/ln(1 + *r*)

 The length of time to double your money is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | ? |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 |  |  |  |  |  |  |  |  |  | $2 |

 FV = $2 = $1(1.0625)*t*

 *t* = ln 2/ln 1.0625 = 11.43 years

 The length of time to quadruple your money is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | ? |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 |  |  |  |  |  |  |  |  |  | $4 |

 FV = $4 = $1(1.0625)*t*

 *t* = ln 4/ln 1.0625

 *t* = 22.87 years

 Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the difference in these answers is due to rounding). This is an important concept of time value of money.

**7.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 20 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV |  |  |  |  |  |  |  |  |  | –$425,000,000 |

 To find the PV of a lump sum, we use:

 PV = FV/(1 + *r)t*

 PV = $425,000,000/(1.059)20

 PV = $135,042,269.46

**8.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 4 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1,680,000 |  |  |  |  |  |  |  |  |  | $1,100,000 |

 To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

 FV = PV(1 + *r*)*t*

 Solving for *r*, we get:

 *r* = (FV/PV)1/*t* – 1

 *r* = ($1,100,000/$1,680,000)1/3 – 1

 *r* = –.1317, or –13.17%

 Notice that the interest rate is negative. This occurs when the FV is less than the PV.

**9.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | ∞ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $75 | $75 | $75 | $75 | $75 | $75 | $75 | $75 | $75 |

 A consol is a perpetuity.To find the PV of a perpetuity, we use the equation:

 PV = *C*/*r*

 PV = $75/.031

 PV = $2,419.35

**10.** To find the future value with continuous compounding, we use the equation:

 FV = PV*ert*

 *a.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 9 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $2,350 |  |  |  |  |  |  |  |  |  | FV |

 FV = $2,350*e*.12(9) = $6,516.74

 *b.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 5 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $2,350 |  |  |  |  |  |  |  |  |  | FV |

 FV = $2,350*e*.08(5) = $3,452.92

 *c.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 17 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $2,350 |  |  |  |  |  |  |  |  |  | FV |

 FV = $2,350*e*.05(17) = $5,386.24

 *d.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $2,350 |  |  |  |  |  |  |  |  |  | FV |

FV *=* $2,350*e*.09(10)= $5,563.30

**11.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $795 | $945 | $1,325 | $1,860 |  |  |  |  |  |  |

 To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

 PV = FV/(1 + *r*)*t*

 PV@10% = $795/1.10 + $945/1.102 + $1,325/1.103 + $1,860/1.104 = $3,769.62

 PV@18% = $795/1.18 + $945/1.182 + $1,325/1.183 + $1,860/1.184 = $3,118.22

 PV@24% = $795/1.24 + $945/1.242 + $1,325/1.243 + $1,860/1.244 = $2,737.40

**12.** The times lines are:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $4,350 | $4,350 | $4,350 | $4,350 | $4,350 | $4,350 | $4,350 | $4,350 | $4,350 |  |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $6,900 | $6,900 | $6,900 | $6,900 | $6,900 |  |  |  |  |  |

 To find the PVA, we use the equation:

 PVA = *C*({1 – [1/(1 + *r)*]*t*}/*r*)

 At an interest rate of 5 percent:

 X@5%: PVA = $4,350{[1 – (1/1.05)9]/.05} = $30,919.02

 Y@5%: PVA = $6,900{[1 – (1/1.05)5]/.05} = $29,873.39

 And at an interest rate of 22 percent:

 X@22%: PVA = $4,350{[1 – (1/1.22)9]/.22} = $16,470.34

 Y@22%: PVA = $6,900{[1 – (1/1.22)5]/.22} = $19,759.11

Notice that the PV of Cash flow X has a greater PV than Cash flow Y at an interest rate of 5 percent, but a lower PV at an interest rate of 22 percent. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger cash flows. At a higher interest rate, these bigger cash flows earlier are more important since the cost of waiting (the interest rate) is so much greater.

**13.** To find the PVA, we use the equation:

 PVA = *C*({1 – [1/(1 + *r)*]*t*}/*r*)

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 15 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 |

 PVA@15 years: PVA = $5,200{[1 – (1/1.07)15]/.07} = $47,361.15

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 40 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 |

 PVA@40 years: PVA = $5,200{[1 – (1/1.07)40]/.07} = $69,324.89

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 75 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 |

 PVA@75 years: PVA = $5,200{[1 – (1/1.07)75]/.07} = $73,821.07

 To find the PV of a perpetuity, we use the equation:

 PV = *C*/*r*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | ∞ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 | $5,200 |

 PV = $5,200/.07

 PV = $74,285.71

 Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75-year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only $464.65.

**14.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | ∞ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 |

 This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

 PV = *C*/*r*

 PV = $15,000/.038

 PV = $394,736.84

 To find the interest rate that equates the perpetuity cash flows with the PV of the cash flows, we can use the PV of a perpetuity equation:

 PV = *C*/*r*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | ∞ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$325,000 | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 | $15,000 |

 $325,000 = $15,000/*r*

 We can now solve for the interest rate as follows:

 *r* = $15,000/$325,000

 *r* = .0462, or 4.62%

**15.** For discrete compounding, to find the EAR, we use the equation:

 EAR = [1 + (APR/*m*)]*m* – 1

 EAR = [1 + (.071/4)]4 – 1 = .0729, or 7.29%

 EAR = [1 + (.132/12)]12 – 1 = .1403, or 14.03%

 EAR = [1 + (.089/365)]365 – 1 = .0931, or 9.31%

 To find the EAR with continuous compounding, we use the equation:

 EAR = *er* – 1

 EAR = *e*.081 – 1 = .0844, or 8.44%

**16.** Here, we are given the EAR and need to find the APR. Using the equation for discrete compounding:

 EAR = [1 + (APR/*m*)]*m* – 1

 We can now solve for the APR. Doing so, we get:

 APR = *m*[(1 + EAR)1/*m* – 1]

 EAR = .101 = [1 + (APR/2)]2 – 1 APR = 2[(1.101)1/2 – 1] = .0986, or 9.86%

 EAR = .174 = [1 + (APR/12)]12 – 1 APR = 12[(1.174)1/12 – 1] = .1615, or 16.15%

 EAR = .086 = [1 + (APR/52)]52 – 1 APR = 52[(1.086)1/52 – 1] = .0826, or 8.26%

 Solving the continuous compounding EAR equation:

 EAR = *er* – 1

 We get:

 APR = ln(1 + EAR)

 APR = ln(1 + .113)

 APR = .1071, or 10.71%

**17.** For discrete compounding, to find the EAR, we use the equation:

 EAR = [1 + (APR/*m*)]*m* – 1

 So, for each bank, the EAR is:

 First National: EAR = [1 + (.114/12)]12 – 1 = .1201, or 12.01%

 First United: EAR = [1 + (.116/2)]2 – 1 = .1194, or 11.94%

 A higher APR does not necessarily mean a higher EAR. The number of compounding periods within a year will also affect the EAR.

**18.** The cost of a case of wine is 10 percent less than the cost of 12 individual bottles, so the cost of a case will be:

 Cost of case = (12)($10)(1 – .10)

 Cost of case = $108

 Now, we need to find the interest rate. The cash flows are an annuity due, so:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 12 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$108$10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 |

 PVA = (1 + *r)C*({1 – [1/(1 + *r*)]*t*}/*r*)

 $108 = (1 + *r*)$10({1 – [1/(1 + *r*)12]/*r*)

 Solving for the interest rate, we get:

 *r* = .0198, or 1.98% per week

 So, the APR of this investment is:

 APR = .0198(52)

 APR = 1.0277, or 102.77%

 And the EAR is:

 EAR = (1 + .0198)52 – 1

 EAR = 1.7668, or 176.68%

 The analysis appears to be correct. He really can earn about 177 percent buying wine by the case. The only question left is this: Can you really find a fine bottle of Bordeaux for $10?

**19.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | ? |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$16,450 | $400 | $400 | $400 | $400 | $400 | $400 | $400 | $400 | $400 |

 Here, we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

 PVA = *C*({1 – [1/(1 + *r*)]*t*}/*r*)

 $16,450 = $400{[1 – (1/1.011)*t*]/.011}

 Now, we solve for *t*:

 1/1.011*t* = 1 – [($16,450)(.011)/($400)]

 1.011*t* = 1/.5476 = 1.8261

 *t* = ln 1.8261/ln 1.011

 *t* = 55.04 months

**20.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 1 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $3 |  |  |  |  |  |  |  |  |  | $4 |

 Here, we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

 FV = PV(1 + *r*)

 $4 = $3(1 + *r*)

 *r* = 4/3 – 1 = 33.33% per week

 The interest rate is 33.33% per week. To find the APR, we multiply this rate by the number of weeks in a year, so:

 APR = (52)33.33%

 APR = 1,733.33%

 And using the equation to find the EAR:

 EAR = [1 + (APR/*m*)]*m* – 1

 EAR = [1 + .3333]52 – 1

 EAR = 313,916,515.69%

 *Intermediate*

**21.** To find the FV of a lump sum with discrete compounding, we use:

 FV = PV(1 + *r*)*t*

 *a.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 11 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,000 |  |  |  |  |  |  |  |  |  | FV |

 FV = $1,000(1.089)11 = $2,554.50

 *b.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 22 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,000 |  |  |  |  |  |  |  |  |  | FV |

 FV = $1,000(1 + .089/2)22 = $2,606.07

 *c.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 132 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,000 |  |  |  |  |  |  |  |  |  | FV |

 FV = $1,000(1 + .089/12)132 = $2,652.19

 *d.*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 11 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,000 |  |  |  |  |  |  |  |  |  | FV |

 To find the future value with continuous compounding, we use the equation:

 FV = PV*ert*

 FV = $1,000*e*.089(11) = $2,661.79

 *e.* The future value increases when the compounding period is shorter because interest is earned on previously accrued interest. The shorter the compounding period, the more frequently interest is earned, and the greater the future value, assuming the same stated interest rate.

**22.** The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest paid by First Simple Bank over 10 years will be:

 .053(10) = .53

 First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of $1, or:

 (1 + *r*)10

 Setting the two equal, we get:

 (.053)(10) = (1 + *r*)10 – 1

 *r* = 1.531/10 – 1

 *r* = .0434, or 4.34%

**23.** Although the stock and bond accounts have different interest rates, we can draw one time line, but we need to remember to apply different interest rates. The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  | **...** |  |  | 360 | 361 | **…** |  | 660 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Stock | $850 | $850 | $850 | $850 | $850 | *C* | *C* | *C* |
| Bond | $350 | $350 |  | $350 | $350 | $350 |  |

 We need to find the annuity payment in retirement. Our retirement savings end at the same time the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

Stock account: FVA = $850[{[1 + (.10/12) ]360 – 1}/(.10/12)] = $1,921,414.74

 Bond account: FVA = $350[{[1 + (.06/12) ]360 – 1}/(.06/12)] = $351,580.26

 So, the total amount saved at retirement is:

 $1,921,414.74 + 351,580.26 = $2,272,995.00

 Solving for the withdrawal amount in retirement using the PVA equation gives us:

 PVA = $2,272,995 = *C*[1 – {1/[1 + (.07/12)]300}/(.07/12)]

 *C* = $2,272,995/141.4869

 *C* = $16,065.06 withdrawal per month

**24.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 4 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 |  |  |  |  |  |  |  |  |  | $4 |

 Since we are looking to quadruple our money, the PV and FV are irrelevant as long as the FV is four times as large as the PV. The number of periods is four, the number of quarters per year. So:

 FV = $4 = $1(1 + *r*)(12/3)

 *r* = .4142, or 41.42%

**25.** Here, we need to find the interest rate for two possible investments. Each investment is a lump sum, so:

 G:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 6 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$65,000 |  |  |  |  |  |  |  |  |  | $125,000 |

 PV = $65,000 = $125,000/(1 + *r*)6

 (1 + *r*)6 = $125,000/$65,000

 *r* = 1.9231/6 – 1

 *r* = .1151, or 11.51%

 H:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$65,000 |  |  |  |  |  |  |  |  |  | $205,000 |

 PV = $65,000 = $205,000/(1 + *r*)10

 (1 + *r*)10 = $205,000/$65,000

 *r* = 3.1541/10 – 1

 *r* = .1217, or 12.17%

**26.** This is a growing perpetuity. The present value of a growing perpetuity is:

 PV = C/(*r* – g)

 PV = $175,000/(.097 – .038)

 PV = $2,966,101.69

 It is important to recognize that when dealing with annuities or perpetuities, the present value equation calculates the present value one period before the first payment. In this case, since the first payment is in two years, we have calculated the present value one year from now. To find the value today, we discount this value as a lump sum. Doing so, we find the value of the cash flow stream today is:

 PV = FV/(1 + *r*)*t*

 PV = $2,966,101.69/(1 + .097)1

 PV = $2,703,830.17

**27.** The dividend payments are made quarterly, so we must use the quarterly interest rate. The quarterly interest rate is:

 Quarterly rate = Stated rate/4

 Quarterly rate = .038/4

 Quarterly rate = .0095

 The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | ∞ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $2.25 | $2.25 | $2.25 | $2.25 | $2.25 | $2.25 | $2.25 | $2.25 | $2.25 |

 Using the present value equation for a perpetuity, we find the value today of the dividends paid must be:

 PV = C/*r*

 PV = $2.25/.0095

 PV = $236.84

**28.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | **…** |  | 30 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV |  |  | $7,300 | $7,300 | $7,300 | $7,300 | $7,300 | $7,300 | $7,300 |

 We can use the PVA annuity equation to answer this question. The annuity has 28 payments, not 27 payments. Since there is a payment made in Year 3, the annuity actually begins in Year 2. So, the value of the annuity in Year 2 is:

 PVA = *C*({1 – [1/(1 + *r)*]*t*}/*r*)

 PVA = $7,300({1 – [1/(1 + .07)]28}/.07)

 PVA = $88,600.91

 This is the value of the annuity one period before the first payment, or Year 2. So, the value of the cash flows today is:

 PV = FV/(1 + *r*)*t*

PV = $88,600.91/(1 + .07)2

 PV = $77,387.47

**29.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | **…** |  | 20 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV |  |  |  |  |  | $750 | $750 | $750 | $750 |

 We need to find the present value of an annuity. Using the PVA equation, and the 11 percent interest rate, we get:

 PVA = *C*({1 – [1/(1 + *r)*]*t*}/*r*)

 PVA = $750({1 – [1/(1 + .11)]15}/.11)

 PVA = $5,393.15

 This is the value of the annuity in Year 5, one period before the first payment. Finding the value of this amount today, we find:

 PV = FV/(1 + *r*)*t*

PV = $5,393.15/(1 + .08)5

 PV = $3,670.49

**30.** The amount borrowed is the value of the home times one minus the down payment, or:

 Amount borrowed = $725,000(1 – .20)

 Amount borrowed = $580,000

 The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 360 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $580,000 | *C* | *C* | *C* | *C* | *C* | *C* | *C* | *C* | *C* |

 The monthly payments with a balloon payment loan are calculated assuming a longer amortization schedule, in this case, 30 years. The payments based on a 30-year repayment schedule would be:

 PVA = $580,000 = *C*({1 – [1/(1 + .054/12)]360}/(.054/12))

 *C* = $3,256.88

Now, at Year 8 (Month 96), we need to find the PV of the payments which have not been made. The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 96 | 97 |  |  |  | **…** |  |  |  |  | 360 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $3,256.88 | $3,256.88 | $3,256.88 | $3,256.88 | $3,256.88 | $3,256.88 | $3,256.88 | $3,256.88 | $3,256.88 |

The balloon payment will be:

 PVA = $3,256.88({1 – [1/(1 + .054/12)]22(12)}/(.054/12))

 PVA = $502,540.87

**31.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 12 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $12,400 |  |  |  |  |  |  |  |  |  | FV |

Here, we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts.After the first six months, the balance will be:

FV = $12,400[1 + (.0199/12)]6 = $12,523.89

 This is the balance in six months. The FV in another six months will be:

 FV = $12,523.89[1 + (.18/12)]6 = $13,694.17

 The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the FV. The interest accrued is:

 Interest = $13,694.17 – 12,400 = $1,294.17

**32.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | ∞ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$2,750,000 | $273,000 | $273,000 | $273,000 | $273,000 | $273,000 | $273,000 | $273,000 | $273,000 | $273,000 |

 The company would be indifferent at the interest rate that makes the present value of the cash flows equal to the cost today. Since the cash flows are a perpetuity, we can use the PV of a perpetuity equation. Doing so, we find:

 PV = *C*/*r*

 $2,750,000 = $273,000/*r*

 *r* = $273,000/$2,750,000

 *r* = .0993, or 9.93%

**33.** The company will accept the project if the present value of the increased cash flows is greater than the cost. The cash flows are a growing perpetuity, so the present value is:

 PV = *C*{[1/(*r* – *g*)] – [1/(*r* – *g*)] × [(1 + *g*)/(1 + *r*)]*t*}

 PV = $41,000{[1/(.10 – .04)] – [1/(.10 – .04)] × [(1 + .04)/(1 + .10)]5}

 PV = $167,112.08

 The company should accept the project since the cost is less than the increased cash flows.

**34.** Since your salary grows at 3.7 percent per year, your salary next year will be:

 Next year’s salary = $74,500(1 + .037)

 Next year’s salary = $77,256.50

 This means your deposit next year will be:

 Next year’s deposit = $77,256.50(.05)

 Next year’s deposit = $3,862.83

 Since your salary grows at 3.7 percent, your deposit will also grow at 3.7 percent. We can use the present value of a growing annuity equation to find the value of your deposits today. Doing so, we find:

 PV = *C*{[1/(*r* – *g*)] – [1/(*r* – *g*)] × [(1 + *g*)/(1 + *r*)]*t*}

 PV = $3,862.83{[1/(.094 – .037)] – [1/(.094 – .037)] × [(1 + .037)/(1 + .094)]40}

 PV = $59,798.32

 Now, we can find the future value of this lump sum in 40 years. We find:

 FV = PV(1 + *r*)*t*

 FV = $59,798.32(1 + .094)40

 FV = $2,174,612.53

 This is the value of your savings in 40 years.

**35.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 20 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $4,700 | $4,700 | $4,700 | $4,700 | $4,700 | $4,700 | $4,700 | $4,700 | $4,700 |

 The relationship between the PVA and the interest rate is:

 PVA falls as *r* increases, and PVA rises as *r* decreases.

 FVA rises as *r* increases, and FVA falls as *r* decreases.

 The present values of $4,700 per year for 20 years at the various interest rates given are:

 PVA@10% = $4,700{[1 – (1/1.10)20]/.10} = $40,013.75

 PVA@5% = $4,700{[1 – (1/1.05)20]/.05} = $58,572.39

 PVA@15% = $4,700{[1 – (1/1.15)20]/.15} = $29,418.86

**36.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | ? |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | –$40,000 |
|  | $350 | $350 | $350 | $350 |  | $350 | $350 | $350 | $350 | $350 |

 Here, we are given the FVA, the interest rate, and the amount of the annuity. We need to solve for the number of payments. Using the FVA equation:

 FVA = $40,000 = $350[{[1 + (.10/12)]*t* – 1}/(.10/12)]

 Solving for *t*, we get:

 1.00833*t* = 1 + [($40,000)(.10/12)/$350]

 *t* = ln 1.95238/ln 1.00833

 *t* = 80.62 payments

**37.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 60 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$88,000 | $1,725 | $1,725 | $1,725 | $1,725 | $1,725 | $1,725 | $1,725 | $1,725 | $1,725 |

 Here, we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

 PVA = $88,000 = $1,725[{1 – [1/(1 + *r*)]60}/*r*]

 To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

 *r* = .548%

 The APR is the periodic interest rate times the number of periods in the year, so:

 APR = 12(.548%)

 APR = 6.58%

**38.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 360 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $1,025 | $1,025 | $1,025 | $1,025 | $1,025 | $1,025 | $1,025 | $1,025 | $1,025 |

 The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the $1,025 monthly payments is:

 PVA = $1,025[(1 – {1/[1 + (.048/12)]}360)/(.048/12)] = $195,362.62

 The monthly payments of $1,025 will amount to a principal payment of $195,362.62. The amount of principal you will still owe is:

 $275,000 – 195,362.62 = $79,637.38

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 360 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $79,637.38 |  |  |  |  |  |  |  |  | FV |

 This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal, will be:

 Balloon payment = $79,637.38[1 + (.048/12)]360

 Balloon payment = $335,161.06

**39.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$6,700 | $1,400 | ? | $2,300 | $2,700 |  |  |  |  |  |  |

 We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and

subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV of the cash flows we know is:

 PV of Year 1 CF: $1,400/1.071 = $1,307.19

 PV of Year 3 CF: $2,300/1.0713 = $1,872.23

 PV of Year 4 CF: $2,700/1.0714 = $2,052.13

 So, the PV of the missing CF is:

 $6,700 – 1,307.19 – 1,872.23 – 2,052.13 = $1,468.44

 The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount. The value of the missing CF is:

 $1,468.44(1.071)2 = $1,684.37

**40.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1M | $1.335M | $1.67M | $2.005M | $2.34M | $2.675M | $3.01M | $3.345M | $3.68M | $4.015M | $4.35M |

 To solve this problem, we need to find the PV of each lump sum and add them together. It is important to note that the first cash flow of $1 million occurs today, so we do not need to discount that cash flow. The PV of the lottery winnings is:

$1,000,000 + $1,335,000/1.058 + $1,670,000/1.0582 + $2,005,000/1.0583 + $2,340,000/1.0584 + $2,675,000/1.0585 + $3,010,000/1.0586 + $3,345,000/1.0587 + $3,680,000/1.0588

 + $4,015,000/1.0589 + $4,350,000/1.05810

 PV = $20,969,067.06

**41.** Here, we are finding the interest rate for an annuity cash flow.We are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. We should also note that the PV of the annuity is not the amount borrowed since we are making a down payment on the warehouse. The amount borrowed is:

 Amount borrowed = .80($5,500,000) = $4,400,000

 The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 360 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$4,400,000 | $26,500 | $26,500 | $26,500 | $26,500 | $26,500 | $26,500 | $26,500 | $26,500 | $26,500 |

 Using the PVA equation:

 PVA = $4,400,000 = $26,500[{1 – [1/(1 + *r*)]360}/*r*]

 Unfortunately, this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

 *r* = .504%

 The APR is the monthly interest rate times the number of months in the year, so:

 APR = 12(.504%)

 APR = 6.04%

 And the EAR is:

 EAR = (1 + .00504)12 – 1

 EAR = .0621, or 6.21%

**42.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 3 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV |  |  |  |  |  |  |  |  |  | $165,000 |

 The profit the firm earns is the PV of the sales price minus the cost to produce the asset. We find the PV of the sales price as the PV of a lump sum:

 PV = $165,000/1.133

 PV = $114,353.28

 And the firm’s profit is:

 Profit = $114,353.28 – 103,000

 Profit = $11,353.28

 To find the interest rate at which the firm will break even, we need to find the interest rate using the PV (or FV) of a lump sum. Using the PV equation for a lump sum, we get:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  |  |  |  |  |  |  |  |  | 3 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$103,000 |  |  |  |  |  |  |  |  |  | $165,000 |

 $103,000 = $165,000/( 1 + *r*)3

 *r* =($165,000/$103,000)1/3 – 1

 *r* = .1701, or 17.01%

**43.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  1 |  | **…** |  | 5 | 6 |  | **…** |  | 25 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | $8,500 | $8,500 | $8,500 | $8,500 |

 We want to find the value of the cash flows today, so we will find the PV of the annuity, and then bring the lump sum PV back to today. The annuity has 20 payments, so the PV of the annuity is:

 PVA = $8,500{[1 – (1/1.067)20]/.067}

 PVA = $92,187.54

 Since this is an ordinary annuity equation, this is the PV one period before the first payment, so it is the PV at *t* = 5. To find the value today, we find the PV of this lump sum. The value today is:

 PV = $92,187.54/1.0675

 PV = $66,657.67

**44.** The time line for the annuity is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 180 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $1,940 | $1,940 | $1,940 | $1,940 | $1,940 | $1,940 | $1,940 | $1,940 | $1,940 |

 This question is asking for the present value of an annuity, but the interest rate changes during the life of the annuity. We need to find the present value of the cash flows for the last eight years first. The PV of these cash flows is:

 PVA2 = $1,940[{1 – 1/[1 + (.06/12)]96}/(.06/12)]

 PVA2 = $147,624.72

Note that this is the PV of this annuity exactly seven years from today. Now, we can discount this lump sum to today as well as finding the PV of the annuity for the first seven years. The value of this cash flow today is:

 PV = $147,624.72/[1 + (.11/12)]84 + $1,940[{1 – 1/[1 + (.11/12)]84}/(.11/12)]

 PV = $181,893.99

**45.** The time line for the annuity is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  1 |  |  |  | **…** |  |  |  |  | 180 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $1,175 | $1,175 | $1,175 | $1,175 | $1,175 | $1,175 | $1,175 | $1,175 | $1,175 |
|  |  |  |  |  |  |  |  |  |  | FV |

 Here, we are trying to find the dollar amount invested today that will equal the FVA with a known interest rate and payments. First, we need to determine how much we would have in the annuity account. Finding the FV of the annuity, we get:

 FVA = $1,175[{[ 1 + (.064/12)]180 – 1}/(.064/12)]

 FVA = $353,610.97

 Now, we need to find the PV of a lump sum that will give us the same FV. So, using the FV of a lump sum with continuous compounding, we get:

 FV = $353,610.97 = PV*e*.07(15)

 PV = $353,610.97*e–*1.05

 PV = $123,741.83

**46.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1  | **…** | 7 | **…** | 14 | 15 |  | **…** |  | ∞ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | PV |  | $2,350 | $2,350 | $2,350 | $2,350 |

 To find the value of the perpetuity at *t* = 7, we first need to use the PV of a perpetuity equation. Using this equation we find:

 PV = $2,350/.063

 PV = $37,301.59

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  | **…** |  | 7 |  |  | **…** |  | 14 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | PV |  |  |  | $37,301.59 |

 Remember that the PV of a perpetuity (and annuity) equation gives the PV one period before the first payment, so, this is the value of the perpetuity at *t* = 14. To find the value at *t* = 7, we find the PV of this lump sum as:

 PV = $37,301.59/1.0637

 PV = $24,321.73

**47.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  |  | 12 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$26,000 | $2,519.83 | $2,519.83 | $2,519.83 | $2,519.83 | $2,519.83 | $2,519.83 | $2,519.83 | $2,519.83 | $2,519.83 |

 To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The interest rate for the cash flows of the loan is:

 PVA = $26,000 = $2,519.83{(1 – [1/(1 + *r*)]12 )/*r*}

 Again, we cannot solve this equation for *r*, so we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. Using a spreadsheet, we find:

 *r* = 2.403% per month

 So the APR is:

 APR = 12(2.403%)

 APR = 28.84%

 And the EAR is:

 EAR = (1.02403)12 – 1

 EAR = 32.97%

**48.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | **…** |  | **…** | 18 | 19 |  | **…** |  | 28 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | $5,230 | $5,230 | $5,230 | $5,230 |

 The cash flows in this problem are semiannual, so we need the effective semiannual rate. The interest rate given is the APR, so the monthly interest rate is:

 Monthly rate = .10/12 = .0083

To get the semiannual interest rate we can use the EAR equation, but instead of using 12 months as the exponent, we will use 6 months. The effective semiannual rate is:

 Semiannual rate = 1.00836 – 1 = .0511, or 5.11%

We can now use this rate to find the PV of the annuity. The PV of the annuity is:

PVA @ *t* = 9: $5,230{[1 – (1/1.0511)10]/.0511} = $40,178.89

Note that this is the value one period (six months) before the first payment, so it is the value at *t* = 9. So, the value at the various times the question asked for uses this value 9 years from now.

 PV @ *t* = 5: $40,178.89/1.05118 = $26,977.40

Note that you can also calculate this present value (as well as the remaining present values) using the number of years. To do this, you need the EAR. The EAR is:

 EAR = (1 + .0083)12 – 1 = .1047, or 10.47%

 So, we can find the PV at *t* = 5 using the following method as well:

 PV @ *t* = 5: $40,178.89/1.10474 = $26,977.40

The value of the annuity at the other times in the problem is:

 PV @ *t* = 3: $40,178.89/1.051112 = $22,105.54

 PV @ *t* = 3: $40,178.89/1.10476 = $22,105.54

 PV @ *t* = 0: $40,178.89/1.051118 = $16,396.55

 PV @ *t* = 0: $40,178.89/1.10479 = $16,396.55

**49.** *a.*  The time line for the ordinary annuity is:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | $13,250 | $13,250 | $13,250 | $13,250 | $13,250 |  |  |  |  |

 If the payments are in the form of an ordinary annuity, the present value will be:

 PVA = *C*({1 – [1/(1 + *r*)*t*]}/*r*))

 PVA = $13,250[{1 – [1/(1 + .078)]5}/.078]

 PVA = $53,183.45

 The time line for the annuity due is:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV |  |  |  |  |  |  |  |  |  |
| $13,250 | $13,250 | $13,250 | $13,250 | $13,250 |  |  |  |  |  |

 If the payments are an annuity due, the present value will be:

 PVAdue = (1 + *r*) PVA

 PVAdue = (1 + .078)$53,183.45

 PVAdue = $57,331.76

 *b.* The time line for the ordinary annuity is:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | FV |  |  |  |  |
|  | $13,250 | $13,250 | $13,250 | $13,250 | $13,250 |  |  |  |  |

 We can find the future value of the ordinary annuity as:

 FVA = *C*{[(1 + *r*)*t* – 1]/*r*}

 FVA = $13,250{[(1 + .078)5 – 1]/.078}

 FVA = $77,423.06

 The time line for the annuity due is:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $13,250 | $13,250 | $13,250 | $13,250 | $13,250 | FV |  |  |  |  |

 If the payments are an annuity due, the future value will be:

 FVAdue = (1 + *r*) FVA

 FVAdue = (1 + .075)$77,423.06

 FVAdue = $83,462.06

 *c.* Assuming a positive interest rate, the present value of an annuity due will always be larger than the present value of an ordinary annuity. Each cash flow in an annuity due is received one period earlier, which means there is one period less to discount each cash flow. Assuming a positive interest rate, the future value of an ordinary due will always be higher than the future value of an ordinary annuity. Since each cash flow is made one period sooner, each cash flow receives one extra period of compounding.

**50.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 |  |  |  | **…** |  |  |  | 59 | 60 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$84,000 |  |  |  |  |  |  |  |  |  |
| *C* | *C* | *C* | *C* | *C* |  | *C* | *C* | *C* | *C* |  |

 We need to use the PVA due equation, that is:

 PVAdue = (1 + *r*)PVA

 Using this equation:

 PVAdue = $84,000 = [1 + (.0608/12)] × *C*[{1 – 1/[1 + (.0608/12)]60}/(.0608/12)

 *C* = $1,618.88

 Notice, to find the payment for the PVA due we compound the payment for an ordinary annuity forward one period.

***CHAPTER 8***

**INTEREST RATES AND BOND VALUATION**

**Answers to Concept Questions**

**1.** No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.

**2.** All else the same, the Treasury security will have lower coupons because of its lower default risk, so it will have greater interest rate risk.

**3.** No. If the bid were higher than the ask, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?

**4.** Prices and yields move in opposite directions. Since the bid price must be lower, the bid yield must be higher.

**5.** Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and determines what the bond’s coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells for exactly par.

**6.** Yes. Some investors have obligations that are denominated in dollars; i.e., they are nominal. Their primary concern is that an investment provides the needed nominal dollar amounts. Pension funds, for example, often must plan for pension payments many years in the future. If those payments are fixed in dollar terms, then it is the nominal return on an investment that is important.

**7.** Companies pay to have their bonds rated because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.

**8.** Treasury bonds have no credit risk since they are backed by the U.S. government, so a rating is unnecessary. Junk bonds often are not rated because there would be no point in an issuer paying a rating agency to assign its bonds a low rating (it’s like paying someone to kick you!).

**9.** The term structure is based on pure discount bonds. The yield curve is based on coupon-bearing issues.

**10.** Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.

**11.** As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other’s securities is referred to as “reciprocal immunity.”

**12.** Lack of transparency means that a buyer or seller can’t see recent transactions, so it is much harder to determine what the best bid and ask prices are at any point in time.

**13.** When the bonds are initially issued, the coupon rate is set at auction so that the bonds sell at par value. The wide range of coupon rates shows the interest rate when each bond was issued. Notice that interest rates have evidently declined. Why?

**14.** Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.

**15.** A 100-year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the “equity in disguise” has a significant tax advantage.

**16.** *a*. The bond price is the present value of the cash flows from a bond. The YTM is the interest rate used in valuing the cash flows from a bond.

 *b*. If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; for discount bonds, the YTM exceeds the coupon rate, and for bonds selling at par, the YTM is equal to the coupon rate.

*c*. Current yield is defined as the annual coupon payment divided by the current bond price. For

premium bonds, the current yield exceeds the YTM, for discount bonds the current yield is less than the YTM, and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.

**17.** A long-term bond has more interest rate risk compared to a short-term bond, all else the same. A low coupon bond has more interest rate risk than a high coupon bond, all else the same. When comparing a high coupon, long-term bond to a low coupon, short-term bond, we are unsure which has more interest rate risk. Generally, the maturity of a bond is a more important determinant of the interest rate risk, so the long-term, high coupon bond probably has more interest rate risk. The exception would be if the maturities are close, and the coupon rates are vastly different.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

 *Basic*

**1.** The price of a pure discount (zero coupon) bond is the present value of the par value. Remember, even though there are no coupon payments, the periods are semiannual to stay consistent with coupon bond payments. So, the price of the bond for each YTM is:

 *a.* P = $1,000/(1 + .06/2)40 = $306.56

 *b.* P = $1,000/(1 + .08/2)40 = $208.29

 *c.* P = $1,000/(1 + .10/2)40 = $142.05

**2.** The price of any bond is the PV of the interest payments, plus the PV of the par value. Notice this problem assumes a semiannual coupon. The price of the bond at each YTM will be:

 *a.* P = $35({1 – [1/(1 + .035)]30}/.035) + $1,000[1/(1 + .035)30]

 P = $1,000.00

 When the YTM and the coupon rate are equal, the bond will sell at par.

 *b.* P = $35({1 – [1/(1 + .045)]30}/.045) + $1,000[1/(1 + .045)30]

 P = $837.11

 When the YTM is greater than the coupon rate, the bond will sell at a discount.

 *c.* P = $35({1 – [1/(1 + .025)]30}/.025) + $1,000[1/(1 + .025)30]

 P = $1,209.30

 When the YTM is less than the coupon rate, the bond will sell at a premium.

**3.** Here we are finding the YTM of a semiannual coupon bond. The bond price equation is:

 P = $980 = $31(PVIFA*R%*,26) + $1,000(PVIF*R%*,26)

 Since we cannot solve the equation directly for *R*, using a spreadsheet, a financial calculator, or trial and error, we find:

 *R* = 3.215%

 Since the coupon payments are semiannual, this is the semiannual interest rate. The YTM is the APR of the bond, so:

 YTM = 2 × 3.215% = 6.43%

**4.** Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

 P = $1,055 = *C*(PVIFA3.4%,23) + $1,000(PVIF3.4%,23)

 Solving for the coupon payment, we get:

 *C* = $37.49

 Since this is the semiannual payment, the annual coupon payment is:

 2 × $37.49 = $74.97

 And the coupon rate is the annual coupon payment divided by par value, so:

 Coupon rate = $74.97/$1,000

 Coupon rate = .0750, or 7.50%

**5.** The price of any bond is the PV of the interest payments, plus the PV of the par value. The fact that the bond is denominated in euros is irrelevant. Notice this problem assumes an annual coupon. The price of the bond will be:

 P = €51({1 – [1/(1 + .043)]15}/.043) + €1,000[1/(1 + .043)15]

 P = €1,087.11

**6.** Here we are finding the YTM of an annual coupon bond. The fact that the bond is denominated in yen is irrelevant. The bond price equation is:

 P = ¥106,000 = ¥2,800(PVIFA*R%*,17) + ¥100,000(PVIF*R%*,17)

 Since we cannot solve the equation directly for *R*, using a spreadsheet, a financial calculator, or trial and error, we find:

 *R* = 2.37%

 Since the coupon payments are annual, this is the yield to maturity.

**7.** To find the price of a zero coupon bond, we need to find the value of the future cash flows. With a zero coupon bond, the only cash flow is the par value at maturity. We find the present value assuming semiannual compounding to keep the YTM of a zero coupon bond equivalent to the YTM of a coupon bond, so:

 P = $10,000(PVIF2.25%,26)

 P = $5,607.30

**8.** To find the price of this bond, we need to find the present value of the bond’s cash flows. So, the price of the bond is:

 P = $49(PVIFA2.70%,26) + $2,000(PVIF2.70%,26)

 P = $1,907.45

**9.** To find the price of this bond, we need to find the present value of the bond’s cash flows. So, the price of the bond is:

 P = $72.50(PVIFA1.70%,24) + $5,000(PVIF1.70%,24)

 P = $4,755.34

**10.** The approximate relationship between nominal interest rates (*R*), real interest rates (*r*), and inflation (*h*) is:

 *R* ≈ *r* + *h*

 Approximate *r* = .0415 – .027

 Approximate *r* = .0145, or 1.45%

 The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

 (1 + *R*) = (1 + *r*)(1 + *h*)

 (1 + .0415) = (1 + *r*)(1 + .027)

 Exact *r* = [(1 + .0415)/(1 + .027)] – 1

 Exact *r* = .0141, or 1.41%

**11.** The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

 (1 + *R*) = (1 + *r*)(1 + *h*)

 *R* = (1 + .0225)(1 + .032) – 1

 *R* = .0552, or 5.52%

**12.** The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

 (1 + *R*) = (1 + *r*)(1 + *h*)

*h* = [(1 + .12)/(1 + .07)] – 1

 *h* = .0467, or 4.67%

**13.** The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

 (1 + *R*) = (1 + *r*)(1 + *h*)

 *r* = [(1 + .1208)/(1.046)] – 1

 *r* = .0715, or 7.15%

**14.** The coupon rate, located in the second column of the quote, is 5.250 percent. The bid price is:

 Bid price = 129.1328 = 129.1328%

 Bid price = (129.1328/100)($10,000)

 Bid price = $12,913.28

 The previous day’s asked price is found by:

 Previous day’s asked price = Today’s asked price – Change = 129.1953 + .1484 = 129.3437

 The previous day’s asked price in dollars was:

 Previous day’s asked price = 129.3437 = 129.3437%

 Previous day’s asked price = (129.3437/100)($10,000)

 Previous day’s asked price = $12,934.37

**15.** This is a premium bond because it sells for more than 100 percent of face value. The dollar asked price is:

 Price = (128.4688/100)($10,000)

 Price = $12,846.88

 The current yield is the annual coupon payment divided by the price, so:

 Current yield = Annual coupon payment/Price

 Current yield = $437.50/$12,846.88

 Current yield = .03405, or 3.405%

 The YTM is located under the “Asked Yield” column, so the YTM is 2.665 percent.

 The bid-ask spread as a percentage of par is:

 Bid-ask spread = 128.4688 – 128.4063

 Bid-ask spread = .0625

 So, in dollars, we get:

 Bid-ask spread = (.0625/100)($10,000)

 Bid-ask spread = $6.25

**16.** Zero coupon bonds are priced with semiannual compounding to correspond with coupon bonds. The price of the bond when purchased was:

 P0 = $1,000/(1 + .0285)40

 P0 = $324.96

 And the price at the end of one year is:

 P0 = $1,000/(1 + .0285)38

 P0 = $343.75

 So, the implied interest, which will be taxable as interest income, is:

 Implied interest = $343.75 – 324.96

 Implied interest = $18.79

 *Intermediate*

**17.** Here we are finding the YTM of annual coupon bonds for various maturity lengths. The bond price equation is:

 P = *C*(PVIFA*R%*,*t*) + $1,000(PVIF*R%*,*t*)

 Bond Miller:

 P0 = $32.50(PVIFA2.65%,26) + $1,000(PVIF2.65%,26) = $1,111.71

 P1 = $32.50(PVIFA2.65%,24) + $1,000(PVIF2.65%,24) = $1,105.55

 P3 = $32.50(PVIFA2.65%,20) + $1,000(PVIF2.65%,20) = $1,092.22

 P8 = $32.50(PVIFA2.65%,10) + $1,000(PVIF2.65%,10) = $1,052.11

 P12 = $32.50(PVIFA2.65%,2) + $1,000(PVIF2.65%,2) = $1,011.54

 P13 = $1,000

 Bond Modigliani:

 P0 = $26.50(PVIFA3.25%,26) + $1,000(PVIF3.25%,26) = $895.76

 P1 = $26.50(PVIFA3.25%,24) + $1,000(PVIF3.25%,24) = $901.07

 P3 = $26.50(PVIFA3.25%,20) + $1,000(PVIF3.25%,20) = $912.76

 P8 = $26.50(PVIFA3.25%,10) + $1,000(PVIF3.25%,10) = $949.47

 P12 = $26.50(PVIFA3.25%,2) + $1,000(PVIF3.25%,2) = $988.56

 P13 = $1,000

 All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called “pull to par.” In both cases, the largest percentage price changes occur at the shortest maturity lengths.

 Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

**18.** Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 5.8 percent. If the YTM suddenly rises to 7.8 percent:

 PLaurel = $29(PVIFA3.9%,6) + $1,000(PVIF3.9%,6) = $947.41

 PHardy = $29(PVIFA3.9%,40) + $1,000(PVIF3.9%,40) = $799.09

 The percentage change in price is calculated as:

 Percentage change in price = (New price – Original price)/Original price

 ΔPLaurel% = ($947.41 – 1,000)/$1,000 = –.0526, or –5.26%

 ΔPHardy% = ($799.09 – 1,000)/$1,000 = –.2009, or –20.09%

 If the YTM suddenly falls to 3.8 percent:

 PLaurel = $29(PVIFA1.9%,6) + $1,000(PVIF1.9%,6) = $1,056.20

 PHardy = $29(PVIFA1.9%,40) + $1,000(PVIF1.9%,40) = $1,278.41

 ΔPLaurel% = ($1,056.20 – 1,000)/$1,000 = .0562, or 5.62%

 ΔPHardy% = ($1,278.41 – 1,000)/$1,000 = .2784, or 27.84%

 All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

**19.** Initially, at a YTM of 9 percent, the prices of the two bonds are:

 PFaulk = $35(PVIFA4.5%,24) + $1,000(PVIF4.5%,24) = $855.05

 PYoo = $55(PVIFA4.5%,24) + $1,000(PVIF4.5%,24) = $1,144.95

 If the YTM rises from 9 percent to 11 percent:

 PFaulk = $35(PVIFA5.5%,24) + $1,000(PVIF5.5%,24) = $736.97

 PYoo = $55(PVIFA5.5%,24) + $1,000(PVIF5.5%,24) = $1,000.00

 The percentage change in price is calculated as:

 Percentage change in price = (New price – Original price)/Original price

 %ΔPFaulk = ($736.97 – 855.05)/$855.05 = –.1381, or –13.81%

 %ΔPYoo = ($1,000.00 – 1,144.95)/$1,144.95 = –.1266, or –12.66%

 If the YTM declines from 9 percent to 7 percent:

 PFaulk = $35(PVIFA3.5%,24) + $1,000(PVIF3.5%,24) = $1,000.00

 PYoo = $55(PVIFA3.5%,24) + $1,000(PVIF3.5%,24) = $1,321.17

 %ΔPFaulk = ($1,000.00 – 855.05)/$855.05 = .1695, or 16.95%

 %ΔPYoo = ($1,321.17 – 1,144.95)/$1,144.95 = .1539, or 15.39%

 All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

**20.** The bond price equation for this bond is:

 P0 = $1,040 = $29.50(PVIFA*R%*,26) + $1,000(PVIF*R%*,26)

 Using a spreadsheet, financial calculator, or trial and error we find:

 *R* = 2.733%

 This is the semiannual interest rate, so the YTM is:

 YTM = 2 × 2.733% = 5.47%

 The current yield is:

 Current yield = Annual coupon payment/Price

 Current yield = $59/$1,040

 Current yield = .0567, or 5.67%

 The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

 Effective annual yield = (1 + .02733)2 – 1

 Effective annual yield = .0554, or 5.54%

**21.** The company should set the coupon rate on its new bonds equal to the required return. The required return can be observed in the market by finding the YTM on outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:

 P = $1,048 = $28.50(PVIFA*R%*,40) + $1,000(PVIF*R%*,40)

 Using a spreadsheet, financial calculator, or trial and error we find:

 *R* = 2.654%

 This is the semiannual interest rate, so the YTM is:

 YTM = 2 × 2.654% = 5.31%

**22.** Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are two months until the next coupon payment, so four months have passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = $64/2 × 4/6

 Accrued interest = $21.33

 And we calculate the clean price as:

 Clean price = Dirty price – Accrued interest

 Clean price = $943 – 21.33

 Clean price = $921.67

**23.** Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are four months until the next coupon payment, so two months have passed since the last coupon payment. The accrued interest for the bond is:

 Accrued interest = $71/2 × 2/6

 Accrued interest = $11.83

 And we calculate the dirty price as:

 Dirty price = Clean price + Accrued interest

 Dirty price = $992 + 11.83

 Dirty price = $1,003.83

***CHAPTER 9***

**STOCK VALUATION**

# Answers to Concept Questions

**1.** The value of any investment depends on the present value of its cash flows; i.e., what investors will actually receive. The cash flows from a share of stock are the dividends.

**2.** Investors believe the company will eventually start paying dividends (or be sold to another company).

**3.** In general, companies that need the cash will often forgo dividends since dividends are a cash expense. Young, growing companies with profitable investment opportunities are one example; another example is a company in financial distress. This question is examined in depth in a later chapter.

**4.** The general method for valuing a share of stock is to find the present value of all expected future dividends. The dividend growth model presented in the text is only valid (i) if dividends are expected to occur forever; that is, the stock provides dividends in perpetuity, and (ii) if a constant growth rate of dividends occurs forever. A violation of the first assumption might be a company that is expected to cease operations and dissolve itself some finite number of years from now. The stock of such a company would be valued by applying the general method of valuation explained in this chapter. A violation of the second assumption might be a start-up firm that isn’t currently paying any dividends, but is expected to eventually start making dividend payments some number of years from now. This stock would also be valued by the general dividend valuation method explained in this chapter.

**5.** The common stock probably has a higher price because the dividend can grow, whereas it is fixed on the preferred. However, the preferred is less risky because of the dividend and liquidation preference, so it is possible the preferred could be worth more, depending on the circumstances.

**6.** The two components are the dividend yield and the capital gains yield. For most companies, the capital gains yield is larger. This is easy to see for companies that pay no dividends. For companies that do pay dividends, the dividend yields are rarely over five percent and are often much less.

**7.** Yes. If the dividend grows at a steady rate, so does the stock price. In other words, the dividend growth rate and the capital gains yield are the same.

**8.** The three factors are: 1) The company’s future growth opportunities. 2) The company’s level of risk, which determines the interest rate used to discount cash flows. 3) The accounting method used.

**9.** It wouldn’t seem to be. Investors who don’t like the voting features of a particular class of stock are under no obligation to buy it.

**10.** Presumably, the current stock value reflects the risk, timing, and magnitude of all future cash flows, both short-term and long-term. If this is correct, then the statement is false.

# Solutions to Questions and Problems

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

 *Basic*

**1.** The constant dividend growth model is:

 P*t* = D*t* × (1 + *g*)/(*R* – *g*)

 So, the price of the stock today is:

 P0 = D0(1 + *g*)/(*R* – *g*) = $2.07(1.043)/(.11 – .043)

 P0 = $32.22

 The dividend at Year 4 is the dividend today times the FVIF for the growth rate in dividends and 4 years, so:

 P3 = D3(1 + *g*)/(*R* – *g*) = D0(1 + g)4/(*R* – *g*)

 P3 = $2.07(1.043)4/(.11 – .043)

 P3 = $36.56

 We can do the same thing to find the dividend in Year 16, which we can use to find the price in Year 15, so:

 P15 = D15(1 + *g*)/(*R* – *g*) = D0(1 + g)16/(*R* – *g*)

 P15 = $2.07(1.043)16/(.11 – .043)

 P15 = $60.60

 There is another feature of the constant dividend growth model: The stock price grows at the dividend growth rate. If we know the stock price today, we can find the future value for any time in the future we want to calculate the stock price. In this problem, we want to know the stock price in 3 years, and we have already calculated the stock price today. The stock price in 3 years will be:

 P3 = P0(1 + *g*)3

 P3 = $32.22(1 + .043)3

 P3 = $36.56

 And the stock price in 15 years will be:

 P15 = P0(1 + *g*)15

 P15 = $32.22(1 + .043)15

 P15 = $60.60

**2.** We need to find the required return of the stock. Using the constant growth model, we can solve the equation for *R*. Doing so, we find:

 *R* = (D1/P0) + *g*

 *R* = ($2.95/$53.10) + .048

 *R* = .1036, or 10.36%

**3.** The dividend yield is the dividend next year divided by the current price, so the dividend yield is:

 Dividend yield = D1/P0

 Dividend yield = $2.95/$53.10

 Dividend yield = .0556, or 5.56%

 The capital gains yield, or percentage increase in the stock price, is the same as the dividend growth rate, so:

 Capital gains yield = 4.8%

**4.** Using the constant growth model, we find the price of the stock today is:

 P0 = D1/(*R* – *g*)

 P0 = $3.25/(.105 – .05)

 P0 = $59.09

**5.** The required return of a stock is made up of two parts: The dividend yield and the capital gains yield. So, the required return of this stock is:

 *R* = Dividend yield + Capital gains yield

 *R* = .049 + .052

 *R* = .1010, or 10.10%

**6.** We know the stock has a required return of 9.9 percent, and the dividend and capital gains yield are equal, so:

 Dividend yield = 1/2(.099) = .0495 = Capital gains yield

 Now we know both the dividend yield and capital gains yield. The dividend is the stock price times the dividend yield, so:

 D1 = .0495($74) = $3.66

 This is the dividend next year. The question asks for the dividend this year. Using the relationship between the dividend this year and the dividend next year:

 D1 = D0(1 + *g*)

 We can solve for the dividend that was just paid:

 $3.66 = D0(1 + .0495)

 D0 = $3.66/1.0495

 D0 = $3.49

**7.** The price of any financial instrument is the PV of the future cash flows. The future dividends of this stock are an annuity for 11 years, so the price of the stock is the PVA, which will be:

 P0 = $8.50(PVIFA9.5%,11)

 P0 = $56.50

**8.** The price of a share of preferred stock is the dividend divided by the required return. This is the same equation as the constant growth model, with a dividend growth rate of zero percent. Remember that most preferred stock pays a fixed dividend, so the growth rate is zero. Using this equation, we find the price per share of the preferred stock is:

 *R* = D/P0

 *R* = $3.75/$81

 *R* = .0463, or 4.63%

**9.** The growth rate of earnings is the return on equity times the retention ratio, so:

 *g* = ROE × *b*

 *g* = .13(.80)

 *g* = .1040, or 10.40%

 To find next year’s earnings, we multiply the current earnings times one plus the growth rate, so:

 Next year’s earnings = Current earnings(1 + *g*)

 Next year’s earnings = $17,500,000(1 + .1040)

 Next year’s earnings = $19,320,000

**10.** Using the equation to calculate the price of a share of stock with the PE ratio:

 P = Benchmark PE ratio × EPS

 So, with a PE ratio of 18, we find:

 P = 18($3.41)

 P = $61.38

 And with a PE ratio of 21, we find:

 P = 21($3.41)

 P = $71.61

 *Intermediate*

**11.** This stock has a constant growth rate of dividends, but the required return changes twice. To find the value of the stock today, we will begin by finding the price of the stock at Year 6, when both the dividend growth rate and the required return are stable forever. The price of the stock in Year 6 will be the dividend in Year 7, divided by the required return minus the growth rate in dividends. So:

 P6 = D6(1 + *g*)/(*R* – *g*) = D0(1 + *g*)7/(*R* – *g*)

 P6 = $3.15(1.04)7/(.11 – .04)

 P6 = $59.22

 Now we can find the price of the stock in Year 3. We need to find the price here since the required return changes at that time. The price of the stock in Year 3 is the PV of the dividends in Years 4, 5, and 6, plus the PV of the stock price in Year 6. The price of the stock in Year 3 is:

 P3 = $3.15(1.04)41.13 + $3.15(1.04)5/1.132 + $3.15(1.04)6/1.133 + $59.22/1.133

 P3 = $50.07

 Finally, we can find the price of the stock today. The price today will be the PV of the dividends in Years 1, 2, and 3, plus the PV of the stock in Year 3. The price of the stock today is:

 P0 = $3.15(1.04)/1.15 + $3.15(1.04)2/(1.15)2 + $3.15(1.04)3/(1.15)3 + $50.07/(1.15)3

 P0 = $40.67

**12.** Here we have a stock that pays no dividends for 9 years. Once the stock begins paying dividends, it will have a constant growth rate of dividends. We can use the constant growth model at that point. It is important to remember that the general form of the constant dividend growth formula is:

 P*t* = [D*t* × (1 + *g*)]/(*R* – *g*)

 This means that since we will use the dividend in Year 10, we will be finding the stock price in Year 9. The dividend growth model is similar to the PVA and the PV of a perpetuity: The equation gives you the PV one period before the first payment. So, the price of the stock in Year 9 will be:

 P9 = D10/(*R* – *g*)

 P9 = $15.75/(.12 – .048)

 P9 = $218.75

 The price of the stock today is the PV of the stock price in the future. We discount the future stock price at the required return. The price of the stock today will be:

 P0 = $218.75/1.129

 P0 = $78.88

**13.** The price of a stock is the PV of the future dividends. This stock is paying five dividends, so the price of the stock is the PV of these dividends using the required return. The price of the stock is:

 P0 = $18/1.11 + $21/1.112 + $24/1.113 + $27/1.114 + $30/1.115

 P0 = $86.40

**14.** With differential dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the differential growth period. The stock begins constant growth in Year 5, so we can find the price of the stock in Year 4, one year before the constant dividend growth begins, as:

 P4 = D4(1 + *g*)/(*R* – *g*) = $2.55(1.045)/(.10 – .045)

 P4 = $48.45

 The price of the stock today is the PV of the first four dividends, plus the PV of the Year 4 stock price. So, the price of the stock today will be:

 P0 = $9/1.10 + $7/1.102 + $5.75/1.103 + ($2.55 + 48.45)/1.104

 P0 = $53.12

**15.** With differential dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the differential growth period. The stock begins constant growth in Year 4, so we can find the price of the stock in Year 3, one year before the constant dividend growth begins as:

 P3 = D3(1 + *g*)/(*R* – *g*) = D0(1 + *g1*)3(1 + *g2*)/(*R* – *g2*)

 P3 = $3.24(1.20)3(1.05)/(.11 – .05)

 P3 = $97.98

 The price of the stock today is the PV of the first three dividends, plus the PV of the Year 3 stock price. The price of the stock today will be:

 P0 = $3.24(1.20)/1.11 + $3.24(1.20)2/1.112 + $3.24(1.20)3/1.113 + $97.98/1.113

 P0 = $83.02

**16.** Here we need to find the dividend next year for a stock experiencing differential growth. We know the stock price, the dividend growth rates, and the required return, but not the dividend. First, we need to realize that the dividend in Year 3 is the current dividend times the FVIF. The dividend in Year 3 will be:

 D3 = D0(1.25)3

 And the dividend in Year 4 will be the dividend in Year 3 times one plus the growth rate, or:

 D4 = D0(1.25)3(1.17)

 The stock begins constant growth after the 4th dividend is paid, so we can find the price of the stock in Year 4 as the dividend in Year 5, divided by the required return minus the growth rate. The equation for the price of the stock in Year 4 is:

 P4 = D4(1 + *g*)/(*R* – *g*)

 Now we can substitute the previous dividend in Year 4 into this equation as follows:

 P4 = D0(1 + *g1*)3(1 + *g2*)(1 + *g3*)/(*R* – *g3*)

 P4 = D0(1.25)3(1.17)(1.05)/(.11 – .05)

 P4 = 39.99D0

 When we solve this equation, we find that the stock price in Year 4 is 39.99 times as large as the dividend today. Now we need to find the equation for the stock price today. The stock price today is the PV of the dividends in Years 1, 2, 3, and 4, plus the PV of the Year 4 price. So:

 P0 = D0(1.25)/1.11 + D0(1.25)2/1.112 + D0(1.25)3/1.113+ D0(1.25)3(1.17)/1.114 + 39.99D0/1.114

 We can factor out D0 in the equation, and combine the last two terms. Doing so, we get:

 P0 = $65.00 = D0{1.25/1.11 + 1.252/1.112 + 1.253/1.113 + [(1.25)3(1.17) + 39.99]/1.114}

 Reducing the equation even further by solving all of the terms in the braces, we get:

 $65 = $31.67D0

 D0 = $65/$31.67

 D0 = $2.05

 This is the dividend today, so the projected dividend for the next year will be:

 D1 = $2.05(1.25)

 D1 = $2.57

**17.** The constant growth model can be applied even if the dividends are declining by a constant percentage, just make sure to recognize the negative growth. So, the price of the stock today will be:

 P0 = D0 (1 + *g*)/(*R* – *g*)

 P0 = $12(1 – .04)/[.09 – (–.04)]

 P0 = $88.62

**18.** We are given the stock price, the dividend growth rate, and the required return, and are asked to find the dividend. Using the constant dividend growth model, we get:

 P0 = $74.32 = D0(1 + *g*)/(*R* – *g*)

 Solving this equation for the dividend gives us:

 D0 = $74.32(.105 – .04)/(1.04)

 D0 = $4.65

**19.** The price of a share of preferred stock is the dividend payment divided by the required return. We know the dividend payment in Year 5, so we can find the price of the stock in Year 4, one year before the first dividend payment. Doing so, we get:

 P4 = $4.00/.043

 P4 = $93.02

 The price of the stock today is the PV of the stock price in the future, so the price today will be:

 P0 = $93.02/(1.043)4

 P0 = $78.61

***CHAPTER 5***

**NET PRESENT VALUE AND OTHER INVESTMENT RULES**

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Assuming conventional cash flows, a payback period less than the project’s life means that the NPV is positive for a zero discount rate, but nothing more definitive can be said. For discount rates greater than zero, the payback period will still be less than the project’s life, but the NPV may be positive, zero, or negative, depending on whether the discount rate is less than, equal to, or greater than the IRR. The discounted payback includes the effect of the relevant discount rate. If a project’s discounted payback period is less than the project’s life, it must be the case that NPV is positive.

**2.** Assuming conventional cash flows, if a project has a positive NPV for a certain discount rate, then it will also have a positive NPV for a zero discount rate; thus, the payback period must be less than the project life. Since discounted payback is calculated at the same discount rate as is NPV, if NPV is positive, the discounted payback period must be less than the project’s life. If NPV is positive, then the present value of future cash inflows is greater than the initial investment cost; thus, PI must be greater than 1. If NPV is positive for a certain discount rate *R*, then it will be zero for some larger discount rate *R*\*; thus, the IRR must be greater than the required return.

**3.** *a.* Payback period is the accounting break-even point of a series of cash flows. To actually compute the payback period, it is assumed that any cash flow occurring during a given period is realized continuously throughout the period, and not at a single point in time. The payback is then the point in time for the series of cash flows when the initial cash outlays are fully recovered. Given some predetermined cutoff for the payback period, the decision rule is to accept projects that pay back before this cutoff, and reject projects that take longer to pay back. The worst problem associated with the payback period is that it ignores the time value of money. In addition, the selection of a hurdle point for the payback period is an arbitrary exercise that lacks any steadfast rule or method. The payback period is biased towards short-term projects; it fully ignores any cash flows that occur after the cutoff point.

 *b.* The IRR is the discount rate that causes the NPV of a series of cash flows to be identically zero. IRR can thus be interpreted as a financial break-even rate of return; at the IRR discount rate, the net value of the project is zero. The acceptance and rejection criteria are:

If *C*0 < 0 and all future cash flows are positive, accept the project if the internal rate of

 return is greater than or equal to the discount rate.

If *C*0 < 0 and all future cash flows are positive, reject the project if the internal rate of

 return is less than the discount rate.

If *C*0 > 0 and all future cash flows are negative, accept the project if the internal rate of

 return is less than or equal to the discount rate.

 If *C*0 > 0 and all future cash flows are negative, reject the project if the internal rate of

 return is greater than the discount rate.

 IRR is the discount rate that causes NPV for a series of cash flows to be zero. NPV is preferred in all situations to IRR; IRR can lead to ambiguous results if there are non-conventional cash flows, and it also may ambiguously rank some mutually exclusive projects. However, for stand-alone projects with conventional cash flows, IRR and NPV are interchangeable techniques.

 *c.* The profitability index is the present value of cash inflows relative to the project cost. As such, it is a benefit/cost ratio, providing a measure of the relative profitability of a project. The profitability index decision rule is to accept projects with a PI greater than one, and to reject projects with a PI less than one. The profitability index can be expressed as: PI = (NPV + cost)/cost = 1 + (NPV/cost). If a firm has a basket of positive NPV projects and is subject to capital rationing, PI may provide a good ranking measure of the projects, indicating the “bang for the buck” of each particular project.

 *d.* NPV is the present value of a project’s cash flows, including the initial outlay. NPV specifically measures, after considering the time value of money, the net increase or decrease in firm wealth due to the project. The decision rule is to accept projects that have a positive NPV, and reject projects with a negative NPV. NPV is superior to the other methods of analysis presented in the text because it has no serious flaws. The method unambiguously ranks mutually exclusive projects, and it can differentiate between projects of different scale and with different time horizons. The only drawback to NPV is that it relies on cash flow and discount rate values that are often estimates and thus not certain, but this is a problem shared by the other performance criteria as well. A project with NPV = $2,500 implies that the total shareholder wealth of the firm will increase by $2,500 if the project is accepted.

**4.** For a project with future cash flows that are an annuity:

 Payback = I/*C*

 And the IRR is:

 0 = – I + *C*/IRR

 Solving the IRR equation for IRR, we get:

 IRR = *C*/I

 Notice this is just the reciprocal of the payback. So:

 IRR = 1/PB

 For long-lived projects with relatively constant cash flows, the sooner the project pays back, the greater is the IRR, and the IRR is approximately equal to the reciprocal of the payback period.

**5.** There are a number of reasons. Two of the most important have to do with transportation costs and exchange rates. Manufacturing in the U.S. places the finished product much closer to the point of sale, resulting in significant savings in transportation costs. It also reduces inventories because goods spend less time in transit. Higher labor costs tend to offset these savings to some degree, at least compared to other possible manufacturing locations. Of great importance is the fact that manufacturing in the U.S. means that a much higher proportion of the costs are paid in dollars. Since sales are in dollars, the net effect is to immunize profits to a large extent against fluctuations in exchange rates. This issue is discussed in greater detail in the chapter on international finance.

**6.** The single biggest difficulty, by far, is coming up with reliable cash flow estimates. Determining an appropriate discount rate is also not a simple task. These issues are discussed in greater depth in the next several chapters. The payback approach is probably the simplest, followed by the AAR, but even these require revenue and cost projections. The discounted cash flow measures (discounted payback, NPV, IRR, and profitability index) are really only slightly more difficult in practice.

**7.** Yes, they are. Such entities generally need to allocate available capital efficiently, just as for-profits do. However, it is frequently the case that the “revenues” from not-for-profit ventures are intangible. For example, charitable giving has real opportunity costs, but the benefits are generally hard to measure. To the extent that benefits are measurable, the question of an appropriate required return remains. Payback rules are commonly used in such cases. Finally, realistic cost/benefit analysis along the lines indicated should definitely be used by the U.S. government and would go a long way toward balancing the budget!

**8.** The statement is false. If the cash flows of Project B occur early and the cash flows of Project A occur late, then, for a low discount rate, the NPV of A can exceed the NPV of B. Observe the following example.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *C*0 | *C*1 | *C*2 | IRR | NPV @ 0% |
| Project A | –$1,000,000 | $0 | $1,440,000 | 20% | $440,000 |
| Project B | –$2,000,000 | $2,400,000 | $0 | 20% | 400,000 |

However, in one particular case, the statement is true for equally risky projects. If the lives of the two projects are equal and the cash flows of Project B are twice the cash flows of Project A in every time period, the NPV of Project B will be twice the NPV of Project A.

**9.** Although the profitability index (PI) is higher for Project B than for Project A, Project A should be chosen because it has the greater NPV. Confusion arises because Project B requires a smaller investment than Project A. Since the denominator of the PI ratio is lower for Project B than for Project A, B can have a higher PI yet have a lower NPV. Only in the case of capital rationing could the company’s decision have been incorrect.

**10.** *a.* Project A would have a higher IRR since the initial investment for Project A is less than that of Project B, if the cash flows for the two projects are identical.

 *b.* Yes, since both the cash flows as well as the initial investment are twice that of Project B.

**11.** Project B’s NPV would be more sensitive to changes in the discount rate. The reason is the time value of money. Cash flows that occur further out in the future are always more sensitive to changes in the interest rate. This sensitivity is similar to the interest rate risk of a bond.

**12.** The MIRR is calculated by finding the present value of all cash outflows, the future value of all cash inflows to the end of the project, and then calculating the IRR of the two cash flows. As a result, the cash flows have been discounted or compounded by one interest rate (the required return), and then the interest rate between the two remaining cash flows is calculated. As such, the MIRR is not a true interest rate. In contrast, consider the IRR. If you take the initial investment, and calculate the future value at the IRR, you can replicate the future cash flows of the project exactly.

**13.** The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the required return, then calculate the NPV of this future value and the initial investment, you will get the same NPV. However, NPV says nothing about reinvestment of intermediate cash flows. The NPV is the present value of the project cash flows. What is actually done with those cash flows once they are generated is irrelevant. Put differently, the value of a project depends on the cash flows generated by the project, not on the future value of those cash flows. The fact that the reinvestment “works” only if you use the required return as the reinvestment rate is also irrelevant because reinvestment is not relevant to the value of the project in the first place.

 One caveat: Our discussion here assumes that the cash flows are truly available once they are generated, meaning that it is up to firm management to decide what to do with the cash flows. In certain cases, there may be a requirement that the cash flows be reinvested. For example, in international investing, a company may be required to reinvest the cash flows in the country in which they are generated and not “repatriate” the money. Such funds are said to be “blocked” and reinvestment becomes relevant because the cash flows are not truly available.

**14.** The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the IRR, then calculate the IRR of this future value and the initial investment, you will get the same IRR. However, as in the previous question, what is done with the cash flows once they are generated does not affect the IRR. Consider the following example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | C0 | C1 | C2 | IRR |
| Project A | –$100 | $10 | $110 | 10% |

 Suppose this $100 is a deposit into a bank account. The IRR of the cash flows is 10 percent. Does the IRR change if the Year 1 cash flow is reinvested in the account, or if it is withdrawn and spent on pizza? No. Finally, consider the yield to maturity calculation on a bond. If you think about it, the YTM is the IRR on the bond, but no mention of a reinvestment assumption for the bond coupons is suggested. The reason is that reinvestment is irrelevant to the YTM calculation; in the same way, reinvestment is irrelevant in the IRR calculation. Our caveat about blocked funds applies here as well.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

 *Basic*

**1.** *a.* The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

 Project A:

 Cumulative cash flows Year 1 = $10,400 = $10,400

 Cumulative cash flows Year 2 = $10,400 + 5,900 = $16,300

 Companies can calculate a more precise value using fractional years. To calculate the fractional payback period, find the fraction of Year 2’s cash flows that is needed for the company to have cumulative undiscounted cash flows of $15,000. Divide the difference between the initial investment and the cumulative undiscounted cash flows as of Year 1 by the undiscounted cash flow of Year 2.

 Payback period = 1 + ($15,000 – 10,400)/$5,900

 Payback period = 1.780 years

 Project B:

 Cumulative cash flows Year 1 = $12,700 = $12,700

 Cumulative cash flows Year 2 = $12,700 + 6,100 = $18,800

 Cumulative cash flows Year 3 = $12,700 + 6,100 + 5,300 = $24,100

 To calculate the fractional payback period, find the fraction of Year 3’s cash flow that is needed for the company to have cumulative undiscounted cash flows of $19,000. Divide the difference between the initial investment and the cumulative undiscounted cash flows as of Year 2 by the undiscounted cash flow of Year 3.

 Payback period = 2 + ($19,000 – 12,700 – 6,100)/$5,300

 Payback period = 2.038 years

 Since Project A has a shorter payback period than Project B has, the company should choose Project A.

 *b.* Discount each project’s cash flows at 15 percent. Choose the project with the highest NPV.

 Project A:

 NPV = –$15,000 + $10,400/1.15 + $5,900/1.152 + $2,100/1.153

 NPV = –$114.49

 Project B:

 NPV = –$19,000 + $12,700/1.15 + $6,100/1.152 + $5,300/1.153

 NPV = $140.79

 The firm should choose Project B since it has a higher NPV than Project A.

**2.** To calculate the payback period, we need to find the time that the project has taken to recover its initial investment. The cash flows in this problem are an annuity, so the calculation is simpler. If the initial cost is $2,700, the payback period is:

 Payback = 4 + ($140/$640) = 4.22 years

 There is a shortcut to calculate the payback period if the future cash flows are an annuity. Divide the initial cost by the annual cash flow. For the $2,700 cost, the payback period is:

 Payback = $2,700/$640 = 4.22 years

 For an initial cost of $3,900, the payback period is:

 Payback = $3,900/$640 = 6.09 years

 The payback period for an initial cost of $6,800 is a little trickier. Notice that the total cash inflows after eight years will be:

 Total cash inflows = 8($640) = $5,120

 If the initial cost is $6,800, the project never pays back. Notice that if you use the shortcut for annuity cash flows, you get:

 Payback = $6,800/$640 = 10.63 years

 This answer does not make sense since the cash flows stop after eight years, so there is no payback period.

**3.** When we use discounted payback, we need to find the value of all cash flows today. The value today of the project cash flows for the first four years is:

 Value today of Year 1 cash flow = $5,000/1.11 = $4,504.50

 Value today of Year 2 cash flow = $5,500/1.112 = $4,463.92

 Value today of Year 3 cash flow = $6,000/1.113 = $4,387.15

 Value today of Year 4 cash flow = $7,000/1.114 = $4,611.12

 To find the discounted payback, we use these values to find the payback period. The discounted first year cash flow is $4,504.50, so the discounted payback for an initial cost of $8,000 is:

 Discounted payback = 1 + ($8,000 – 4,504.50)/$4,463.92 = 1.78 years

 For an initial cost of $12,000, the discounted payback is:

 Discounted payback = 2 + ($12,000 – 4,504.50 – 4,463.92)/$4,387.15 = 2.69 years

 Notice the calculation of discounted payback. We know the payback period is between two and three years, so we subtract the discounted values of the Year 1 and Year 2 cash flows from the initial cost. This is the numerator, which is the discounted amount we still need to make to recover our initial investment. We divide this amount by the discounted amount we will earn in Year 3 to get the fractional portion of the discounted payback.

 If the initial cost is $16,000, the discounted payback is:

 Discounted payback = 3 + ($16,000 – 4,504.50 – 4,463.92 – 4,387.15)/$4,611.12 = 3.57 years

**4.** To calculate the discounted payback, discount all future cash flows back to the present, and use these discounted cash flows to calculate the payback period. To find the fractional year, we divide the amount we need to make in the last year to pay back the project by the amount we will make. Doing so, we find:

 *r* = 0%: 3 + ($3,400/$3,900) = 3.87 years

Discounted payback = Regular payback = 3.87 years

 *r* = 10% $3,900/1.10 + $3,900/1.102 + $3,900/1.103 + $3,900/1.104 *+* $3,900/1.105 = $14,784.07

 $3,900/1.106 = $2,201.45

 Discounted payback = 5 + ($15,100 – 14,784.07)/$2,201.45 = 5.14 years

 *r* = 17%: $3,900/1.17 + $3,900/1.172 + $3,900/1.173 + $3,900/1.174 + $3,900/1.175

+ $3,900/1.176 = $13,997.82

 The project never pays back.

**5.** The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

0 = *C*0 + *C*1/(1 + IRR) + *C*2/(1 + IRR)2 + *C*3/(1 + IRR)3

 0 = –$27,000 + $13,100/(1 + IRR) + $17,200/(1 + IRR)2 + $8,400/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRR = 21.80%

 Since the IRR is greater than the required return we would accept the project.

**6.** The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for Project A is:

0 = *C*0 + *C*1/(1 + IRR) + *C*2/(1 + IRR)2 + *C*3/(1 + IRR)3

 0 = –$7,300 + $3,940/(1 + IRR) + $3,450/(1 + IRR)2 + $2,480/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRR = 18.24%

 And the IRR for Project B is:

0 = *C*0 + *C*1/(1 + IRR) + *C*2/(1 + IRR)2 + *C*3/(1 + IRR)3

 0 = –$4,390 + $2,170/(1 + IRR) + $2,210/(1 + IRR)2 + $1,730/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRR = 19.31%

**7.** The profitability index is defined as the PV of the future cash flows divided by the PV of the initial cost. The cash flows from this project are an annuity, so the equation for the profitability index is:

 PI = *C*(PVIFA*R*,*t*)/*C*0

 PI = $67,000(PVIFA13%,7)/$325,000

 PI = .912

**8.** *a.*The profitability index is the present value of the future cash flows divided by the initial cost. So, for Project Alpha, the profitability index is:

 PIAlpha = [$1,500/1.085 + $1,300/1.0852 + $1,100/1.0853]/$2,700 = 1.240

 And for Project Beta the profitability index is:

PIBeta = [$900/1.085 + $2,600/1.0852 + $3,200/1.0853]/$4,100 = 1.352

 *b.* According to the profitability index, you would accept Project Beta. However, remember the profitability index rule can lead to an incorrect decision when ranking mutually exclusive projects.

 *Intermediate*

**9.** *a.* To have a payback equal to the project’s life, given *C* is a constant cash flow for N years:

 *C* = *I*/*N*

 *b.* To have a positive NPV, *I* < *C* (PVIFA*r*%, *N*). Thus, *C* > *I*/(PVIFA*r*%, *N*).

 *c.* Benefit = *C*(PVIFA*r%, N*) = 2 × costs = 2*I*

 *C* = 2*I*/(PVIFA*r%, N*)

**10.** *a.* The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

 0 = *C*0 + *C*1/(1 + IRR) + *C*2/(1 + IRR)2 + *C*3/(1 + IRR)3 + *C*4/(1 + IRR)4

 0 = $8,700 – $3,900/(1 + IRR) – $2,900/(1 + IRR)2 – $2,300/(1 + IRR)3

 – $1,800/(1 +IRR)4

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRR = 11.21%

 *b.* This problem differs since the initial cash flow is positive and all future cash flows are negative. In other words, this is a financing-type project. For financing projects, accept the project when the IRR is less than the discount rate. Reject the project when the IRR is greater than the discount rate.

 IRR = 11.21%

 Discount Rate = 10%

 IRR > Discount Rate

 Reject the offer when the discount rate is less than the IRR.

 *c.* Using the same reason as part *b*, we would accept the project if the discount rate is 20 percent.

 IRR = 11.21%

 Discount Rate = 20%

 IRR < Discount Rate

 Accept the offer when the discount rate is greater than the IRR.

 *d.* The NPV is the sum of the present value of all cash flows, so the NPV of the project if the discount rate is 10 percent will be:

 NPV = $8,700 – $3,900/1.1 – $2,900/1.12 – $2,300/1.13 – $1,800/1.14

 NPV = –$199.60

 When the discount rate is 10 percent, the NPV of the offer is –$199.60. Reject the offer.

 And the NPV of the project if the discount rate is 20 percent will be:

 NPV = $8,700 – $3,900/1.2 – $2,900/1.22 – $2,300/1.23 – $1,800/1.24

 NPV = $1,237.04

 When the discount rate is 20 percent, the NPV of the offer is $1,237.04. Accept the offer.

 *e.* Yes, the decisions under the NPV rule are consistent with the choices made under the IRR rule since the signs of the cash flows change only once.

**11.** *a.* The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR for each project is:

 Deepwater Fishing IRR:

 0 = *C*0 + *C*1/(1 + IRR) + *C*2/(1 + IRR)2 + *C*3/(1 + IRR)3

 0 = –$725,000 + $270,000/(1 + IRR) + $420,000/(1 + IRR)2 + $380,000/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRR = 20.96%

 Submarine Ride IRR:

 0 = *C*0 + *C*1/(1 + IRR) + *C*2/(1 + IRR)2 + *C*3/(1 + IRR)3

 0 = –$1,450,000 + $820,000/(1 + IRR) + $650,000/(1 + IRR)2 + $540,000/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRR = 19.87%

 Based on the IRR rule, the deepwater fishing project should be chosen because it has the higher IRR.

 *b.* To calculate the incremental IRR, we subtract the smaller project’s cash flows from the larger project’s cash flows. In this case, we subtract the deepwater fishing cash flows from the submarine ride cash flows. The incremental IRR is the IRR of these incremental cash flows. So, the incremental cash flows of the submarine ride are:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year 0 | Year 1 | Year 2 | Year 3 |
| Submarine Ride | –$1,450,000 | $820,000 | $650,000 | $540,000 |
| Deepwater Fishing | –725,000 | 270,000 | 420,000 | 380,000 |
| Submarine – Fishing | –$725,000 | $550,000 | $230,000 | $160,000 |

 Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

 0 = *C*0 + *C*1/(1 + IRR) + *C*2/(1 + IRR)2 + *C*3/(1 + IRR)3

 0 = –$725,000 + $550,000/(1 + IRR) + $230,000/(1 + IRR)2 + $160,000/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 Incremental IRR = 18.40%

 For investing-type projects, accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 18.40 percent, is greater than the required rate of return of 14 percent, choose the submarine ride project. Note that this is not the choice when evaluating only the IRR of each project. The IRR decision rule is flawed because there is a scale problem. That is, the submarine ride has a greater initial investment than does the deepwater fishing project. This problem is corrected by calculating the IRR of the incremental cash flows, or by evaluating the NPV of each project.

 *c.* The NPV is the sum of the present value of the cash flows from the project, so the NPV of each project will be:

 Deepwater Fishing:

 NPV = –$725,000 + $270,000/1.14 + $420,000/1.142 + $380,000/1.143

 NPV = $91,507.64

 Submarine Ride:

 NPV = –$1,450,000 + $820,000/1.14 + $650,000/1.142 + $540,000/1.143

 NPV = $133,936.76

 Since the NPV of the submarine ride project is greater than the NPV of the deepwater fishing project, choose the submarine ride project. The incremental IRR rule is always consistent with the NPV rule.

**12.** *a.* The profitability index is the PV of the future cash flows divided by the initial investment. The cash flows for both projects are an annuity, so:

 PII = $23,200(PVIFA10%,3 )/$45,000 = 1.282

 PIII = $11,700(PVIFA10%,3)/$19,800 = 1.470

 The profitability index decision rule implies that we accept Project II, since PIII is greater than PII.

 *b.* The NPV of each project is:

 NPVI = –$45,000 + $23,200(PVIFA10%,3) = $12,694.97

 NPVII = –$19,800 + $11,700(PVIFA10%,3) = $9,296.17

 The NPV decision rule implies accepting Project I, since NPVI is greater than NPVII.

 *c.* Using the profitability index to compare mutually exclusive projects can be ambiguous when the magnitudes of the cash flows for the two projects are of different scales. In this problem, Project I is more than twice as large as Project II and produces a larger NPV, yet the profitability index criterion implies that Project II is more acceptable.

**13.** *a.* The equation for the NPV of the project is:

 NPV = –$65,000,000 + $92,000,000/1.1 – $11,000,000/1.12

 NPV = $9,545,454.55

 The NPV is greater than 0, so we would accept the project.

 *b.* The equation for the IRR of the project is:

 0 = –$65,000,000 + $92,000,000/(1 + IRR) – $11,000,000/(1 + IRR)2

 From Descartes’ rule of signs, we know there are two IRRs since the cash flows change signs twice. From trial and error, the two IRRs are:

 IRR = 28.35%, –86.82%

 When there are multiple IRRs, the IRR decision rule is ambiguous. Both IRRs are correct; that is, both discount rates make the NPV of the project equal to zero. If we are evaluating whether or not to accept this project, we would not want to use the IRR to make our decision.

**14.** *a.* The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

 Board game:

 Cumulative cash flows Year 1 = $670 = $670

 Cumulative cash flows Year 2 = $670 + 510 = $1,180

 Payback period = 1 + ($850 – 670)/$510 = 1.35 years

 DVD:

 Cumulative cash flows Year 1 = $1,300 = $1,300

 Cumulative cash flows Year 2 = $1,300 + 750 = $2,050

 Payback period = 1 + ($1,700 – 1,300)/$750

 Payback period = 1.53 years

 Since the board game has a shorter payback period than the DVD project, the company should choose the board game.

 *b.* The NPV is the sum of the present value of the cash flows from the project, so the NPV of each project will be:

 Board game:

 NPV = –$850 + $670/1.10 + $510/1.102 + $90/1.103

 NPV = $248.20

 DVD:

 NPV = –$1,700 + $1,300/1.10 + $750/1.102 + $350/1.103

 NPV = $364.61

 Since the NPV of the DVD is greater than the NPV of the board game, choose the DVD.

 *c.* The IRR is the interest rate that makes the NPV of a project equal to zero. So, the IRR of each project is:

 Board game:

 0 = –$850 + $670/(1 + IRR) + $510/(1 + IRR)2 + $90/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRR = 30.86%

 DVD:

 0 = –$1,700 + $1,300/(1 + IRR) + $750/(1 + IRR)2 + $350/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRR = 24.96%

 Since the IRR of the board game is greater than the IRR of the DVD, IRR implies we choose the board game. Note that this is the choice when evaluating only the IRR of each project. The IRR decision rule is flawed because there is a scale problem. That is, the DVD has a greater initial investment than does the board game. This problem is corrected by calculating the IRR of the incremental cash flows, or by evaluating the NPV of each project.

 *d.* To calculate the incremental IRR, we subtract the smaller project’s cash flows from the larger project’s cash flows. In this case, we subtract the board game cash flows from the DVD cash flows. The incremental IRR is the IRR of these incremental cash flows. So, the incremental cash flows of the DVD are:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year 0 | Year 1 | Year 2 | Year 3 |
| DVD | –$1,700 | $1,300 | $750 | $350 |
| Board game | –850 | 670 | 510 | 90 |
| DVD – Board game  | –$850 | $630 | $240 | $260 |

 Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

 0 = *C*0 + *C*1/(1 + IRR) + *C*2/(1 + IRR)2 + *C*3/(1 + IRR)3

 0 = –$850 + $630/(1 + IRR) + $240/(1 + IRR)2 + $260/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 Incremental IRR = 19.29%

 For investing-type projects, accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 19.29 percent, is greater than the required rate of return of 10 percent, choose the DVD project.

**15.** *a.* The profitability index is the PV of the future cash flows divided by the initial investment. The profitability index for each project is:

 PICDMA = [$23,000,000/1.10 + $16,000,000/1.102 + $6,000,000/1.103]/$18,000,000 = 2.15

 PIG4 = [$21,000,000/1.10 + $51,000,000/1.102 + $41,000,000/1.103]/$25,000,000 = 3.68

 PIWi-Fi = [$39,000,000/1.10 + $66,000,000/1.102 + $42,000,000/1.103]/$43,000,000 = 2.83

 The profitability index implies we accept the G4 project. Remember this is not necessarily correct because the profitability index does not necessarily rank projects with different initial investments correctly.

 *b.* The NPV of each project is:

 NPVCDMA = –$18,000,000 + $23,000,000/1.10 + $16,000,000/1.102 + $6,000,000/1.103

 NPVCDMA = $20,640,120.21

 NPVG4 = –$25,000,000 + $21,000,000/1.10 + $51,000,000/1.102 + $41,000,000/1.103

 NPVG4 = $67,043,576.26

 NPVWi-Fi = –$43,000,000 + $39,000,000/1.10 + $66,000,000/1.102 + $42,000,000/1.103

 NPVWi-Fi = $78,555,221.64

 NPV implies we accept the Wi-Fi project since it has the highest NPV. This is the correct decision if the projects are mutually exclusive.

 *c.* We would like to invest in all three projects since each has a positive NPV. If the budget is limited to $43 million, we can only accept the CDMA project and the G4 project, or the Wi-Fi project. NPV is additive across projects and the company. The total NPV of the CDMA project and the G4 project is:

 NPVCDMA and G4 = $20,640,120.21 + 67,043,576.26

 NPVCDMA and G4 = $87,683,696.47

 This is greater than the Wi-Fi project, so we should accept the CDMA project and the G4 project.

**16.** *a.* The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

 AZM Mini-SUV:

 Cumulative cash flows Year 1 = $373,000 = $373,000

 Cumulative cash flows Year 2 = $373,000 + 219,000 = $592,000

 Payback period = 1 + $202,000/$219,000 = 1.92 years

 AZF Full-SUV:

 Cumulative cash flows Year 1 = $395,000 = $395,000

 Cumulative cash flows Year 2 = $395,000 + 477,000 = $872,000

 Cumulative cash flows Year 2 = $395,000 + 477,000 + 339,000 = $1,211,000

 Payback period = 2 + $108,000/$339,000 = 2.32 years

 Since the AZM has a shorter payback period than the AZF, the company should choose the AZM. Remember the payback period does not necessarily rank projects correctly.

 *b.* The NPV of each project is:

 NPVAZM = –$575,000 + $373,000/1.10 + $219,000/1.102 + $185,000/1.103

 NPVAZM = $84,075.88

 NPVAZF = –$980,000 + $395,000/1.10 + $477,000/1.102 + $339,000/1.103

 NPVAZF = $28,001.50

 The NPV criteria implies we accept the AZM because it has the highest NPV.

 *c.* The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the AZM is:

 0 = –$575,000 + $373,000/(1 + IRR) + $219,000/(1 + IRR)2 + $185,000/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRRAZM = 19.36%

 And the IRR of the AZF is:

 0 = –$980,000 + $395,000/(1 + IRR) + $477,000/(1 + IRR)2 + $339,000/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRRAZF = 11.65%

The IRR criteria implies we accept the AZM because it has the highest IRR. Remember the IRR does not necessarily rank projects correctly.

*d.* Incremental IRR analysis is not necessary. The AZM has the smallest initial investment, and the largest NPV, so it should be accepted.

**17.** *a.* The profitability index is the PV of the future cash flows divided by the initial investment. The profitability index for each project is:

 PIA = [$165,000/1.12 + $165,000/1.122]/$225,000 = 1.24

 PIB = [$300,000/1.12 + $300,000/1.122]/$450,000 = 1.13

 PIC = [$180,000/1.12 + $135,000/1.122]/$225,000 = 1.19

 *b.* The NPV of each project is:

 NPVA = –$225,000 + $165,000/1.12 + $165,000/1.122

 NPVA = $53,858.42

 NPVB = –$450,000 + $300,000/1.12 + $300,000/1.122

 NPVB = $57,015.31

 NPVC = –$225,000 + $180,000/1.12 + $135,000/1.122

 NPVC = $43,335.46

 *c.* Accept Projects A, B, and C. Since the projects are independent, accept all three projects because the respective profitability index of each is greater than 1.

 *d.* Accept Project B. Since the Projects are mutually exclusive, choose the Project with the highest PI, while taking into account the scale of the Project. Because Projects A and C have the same initial investment, the problem of scale does not arise when comparing the profitability indexes. Based on the profitability index rule, Project C can be eliminated because its PI is less than the PI of Project A. Because of the problem of scale, we cannot compare the PIs of Projects A and B. However, we can calculate the PI of the incremental cash flows of the two projects, which are:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Project | *C*0 | *C*1 | *C*2 |
|  | B – A | –$225,000 | $135,000 | $135,000 |

 When calculating incremental cash flows, remember to subtract the cash flows of the project with the smaller initial cash outflow from those of the project with the larger initial cash outflow. This procedure insures that the incremental initial cash outflow will be negative. The incremental PI calculation is:

 PI(B – A) = [$135,000/1.12 + $135,000/1.122]/$225,000

 PI(B – A) = 1.014

 The company should accept Project B since the PI of the incremental cash flows is greater than 1.

 *e.* Remember that the NPV is additive across projects. Since we can spend $450,000, we could take Project B or Project A and Project C. Since the combined NPV of Projects A and C is higher than Project B, we should take both Projects A and C.

**18.** *a.* The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

 Dry Prepeg:

 Cumulative cash flows Year 1 = $900,000 = $900,000

 Cumulative cash flows Year 2 = $900,000 + 700,000 = $1,600,000

 Payback period = 1 + ($600,000/$700,000) = 1.86 years

 Solvent Prepeg:

 Cumulative cash flows Year 1 = $345,000 = $345,000

 Cumulative cash flows Year 2 = $345,000 + 570,000 = $915,000

 Payback period = 1 + ($305,000/$570,000) = 1.54 years

 Since the solvent prepeg has a shorter payback period than the dry prepeg, the company should choose the solvent prepeg. Remember the payback period does not necessarily rank projects correctly.

 *b.* The NPV of each project is:

 NPVDry prepeg = –$1,500,000 + $900,000/1.08 + $700,000/1.082 + $725,000/1.083

 NPVDry prepeg = $508,998.88

 NPVSolvent perpeg = –$650,000 + $345,000/1.08 + $570,000/1.082 + $360,000/1.083

 NPVSolvent prepeg = $443,907.18

 The NPV criteria implies accepting the dry prepeg because it has the highest NPV.

 *c.* The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the dry prepeg is:

 0 = –$1,500,000 + $900,000/(1 + IRR) + $700,000/(1 + IRR)2 + $725,000/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRRDry prepeg = 26.84%

 And the IRR of the solvent prepeg is:

 0 = –$650,000 + $345,000/(1 + IRR) + $570,000/(1 + IRR)2 + $360,000/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 IRRSolvent prepeg = 42.16%

The IRR criteria implies accepting the solvent prepeg because it has the highest IRR. Remember the IRR does not necessarily rank projects correctly.

*d.* Incremental IRR analysis is necessary. The solvent prepeg has a higher IRR, but is relatively smaller in terms of investment and NPV. In calculating the incremental cash flows, we subtract the cash flows from the project with the smaller initial investment from the cash flows of the project with the large initial investment, so the incremental cash flows are:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year 0 | Year 1 | Year 2 | Year 3 |
| Dry prepeg | –$1,500,000 | $900,000 | $700,000 | $725,000 |
| Solvent prepeg | –650,000 | 345,000 | 570,000 | 360,000 |
| Dry prepeg – Solvent prepeg  | –$ 850,000 | $555,000 | $130,000 | $365,000 |

 Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

 0 = –$850,000 + $555,000/(1 + IRR) + $130,000/(1 + IRR)2 + $365,000/(1 + IRR)3

 Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

 Incremental IRR = 12.68%

 For investing-type projects, we accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 12.68%, is greater than the required rate of return of 8 percent, we choose the dry prepeg.